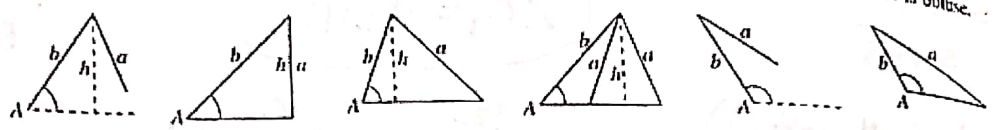
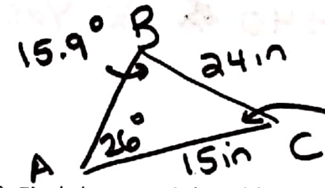
The Ambiguous Case (SSA)

Consider a triangle in which you are given a, b, and A. ($h = b \sin A$)

	A is acute.	A is acute.	A is acute.	A is acute.	A is obtuse.	A is obtuse.
Sketch						
Necessary condition	$a < h$	$a = h$	$a \geq b$	$h < a < b$	$a \leq b$	$a > b$
Triangles possible	None	One	One	Two	None	One



No rounding until final ANS!



Example #3 Single-Solution Case - SSA

use long #s in calc

- For a triangle with a = 24 inches, b = 15 inches, and $A = 26°$. Find the remaining side and angles.

$\frac{24}{\sin 26} = \frac{15}{\sin B}$

$\frac{24 \sin B}{24} = \frac{15 \sin 26}{24}$

$\sin B =$

$B = \sin^{-1}\left(\frac{15\sin(26)}{24}\right)$

$\angle B = 15.90°$

or

$\angle B_2 = 164.10° + 26 > 180$ No 2nd Δ

$m\angle C = 138.10°$

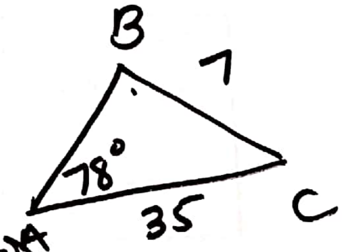
$\frac{24}{\sin 26} = \frac{c}{\sin(138.10)}$

$c = \frac{24\sin(138.1)}{\sin(26)}$

$c = 36.56$ in

$\angle B = 15.9°$
$\angle C = 138.1°$
$c = 36.56$ in

You try: Given $A = 31°$, a = 12, and b = 5, find the remaining side and angles of the triangle.



Example 4 No-Solution – SSA

Show that that there is no triangle for which $A = 78°$, a = 7, and b = 35.

$\frac{7}{\sin 78} = \frac{35}{\sin B}$

$\frac{35 \sin 78}{7} = \frac{7 \sin B}{7}$

$B = \sin^{-1}\left(\frac{35\sin(78)}{7}\right)$ ← 4.89.... not betw. −1 & 1

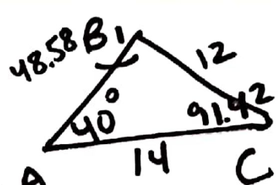
No soln

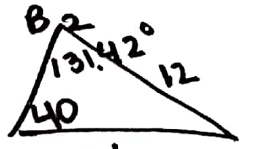
No Δ

You try:

Show that there is no triangle for which $A = 85°$, a = 15, and b = 25.

Example 5 Two – Solution Case - SSA





Find two triangles for which $A = 40°$, ~~$a = 12$, and $b = 14$~~.

$$\frac{12}{\sin 40} = \frac{14}{\sin B}$$

$$B = \sin^{-1}\left(\frac{14\sin(40)}{12}\right)$$

$$B_1 = 48.58°$$

$$B_2 = 131.42° + 40 < 180 \text{ Yes 2nd } \Delta$$

$$\angle C_1 = 91.42°$$

$$\frac{12}{\sin 40} = \frac{c}{\sin 91.42}$$

$$c = \frac{12\sin(91.42)}{\sin 40}$$

$$c_1 = 18.66$$

$$B_2 = 131.42°$$

$$C_2 = 8.58°$$

$$\frac{12}{\sin 40} = \frac{c}{\sin 8.58}$$

$$c = \frac{12\sin(8.58)}{\sin(40)}$$

$$c_2 = 2.79$$

You try:

Find two triangles for which $A = 58°$, $a = 4.5$, and $b = 5$.

Area of an Oblique Triangle

> **Area of an Oblique Triangle**
>
> The area of any triangle is one-half the product of the lengths of two sides times the sine of their included angle. That is,
>
> $$\text{Area} = \frac{1}{2}bc\sin A = \frac{1}{2}ab\sin C = \frac{1}{2}ac\sin B.$$

Example 6: Finding the Area of a Triangular Lot

Find the area of a triangular lot containing side lengths that measure 84 yards and 55 yards and form an angle of 115°.

You try:

Find the area of a triangular lot having two sides of lengths 24 inches and 18 inches and an included angle of 80°.